Lab 3. Static Magnetic Fields

Name: ___________________________

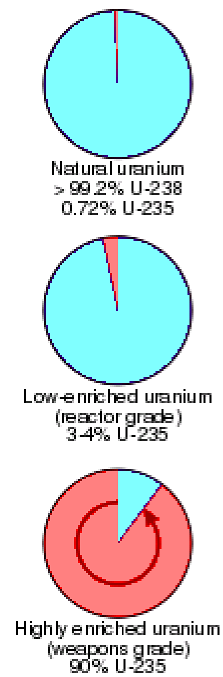
Section: ___________________________

Task 5. The Nuclear Inspector

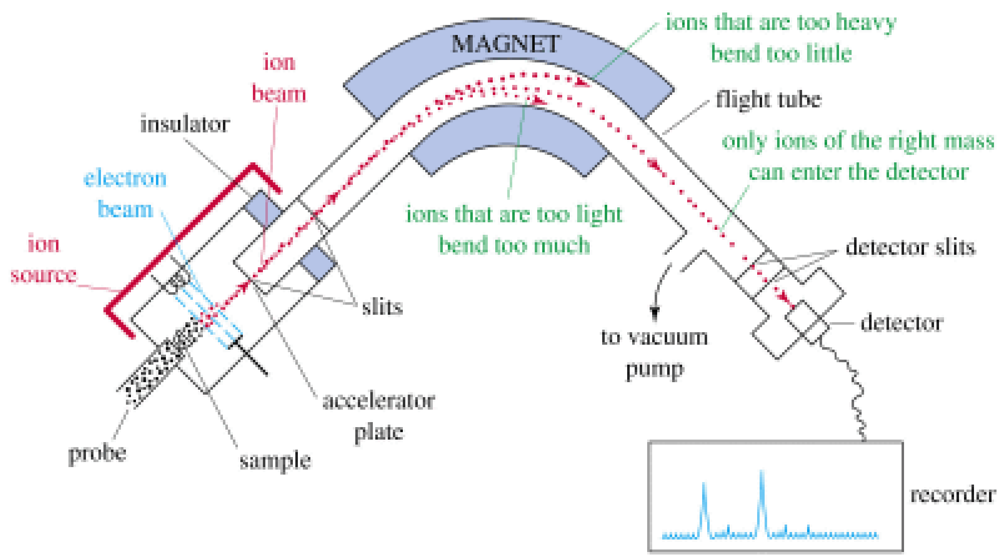
Uranium is a chemical element with 92 protons. It can have between 141 and 146 neutrons. Thus, the atomic mass of the uranium isotopes ranges between 233 and 238.

In nature, uranium atoms exist as uranium-238 (99.284%), uranium-235 (0.711%), and a very small amount of uranium-234 (0.0058%).

Many contemporary uses of uranium exploit its unique nuclear properties. Uranium-235 has the distinction of being the only naturally occurring fissile isotope. While uranium-238 has a small probability to fission spontaneously or when bombarded with fast neutrons, the much higher probability of uranium-235 to fission when bombarded with slow neutrons generates the heat in nuclear reactors. However, it is not only used as a source of power, but also provides the fissile material for nuclear weapons. In reactor-grade uranium, the U-235 fraction is increased to a level of 3–4%, whereas in weapons-grade uranium U-235 is at a level of ~90%.



You are a member of the International Atomic Energy Agency (IAEA) and have been assigned to visit a nuclear facility of a nation that is suspected of enriching uranium 235 for the purpose of making a nuclear weapon. They claim that they are pursuing only peaceful uses of nuclear material, but you suspect otherwise. After much negotiating, your team has been granted access to their enrichment facility and you have been sent to inspect the mass spectrometer measurement system. The mass spectrometer is used for periodical checks of samples out of the gas centrifuge system to see how enriched the uranium is. A schematic of the mass spec can be seen below.



A Java applet showing the operation of a mass spec may be found at
http://www.magnet.fsu.edu/education/tutorials/java/singlesector2/index.html .

Rather than a magnet as shown above, the magnetic field is generated by a large coil producing uniform magnetic field perpendicular to the plane of the drawing. The material to be examined is bombarded with electrons and ionized in the ion source chamber. Then the ions are accelerated in a strong electric field, by applying high voltage **(set by the operator**) to the accelerator's plate. The charged particles' trajectories are later altered by the magnetic field so that only one particular isotope reaches the detector (assuming all ions begin at rest and are singly ionized, i.e., only one electron has been knocked off the atom).

In order to measure a few samples and see if they are weapon-grade, you need the calibration curve of the mass spectrometer. However, you could not find the curve among the instrument documentation, so you decide to figure it out from the available data.

The calibration curve gives the dependence of the acceleration voltage on the ion's atomic mass. It allows the operator to set a proper acceleration voltage to measure the amount of given isotope in the probe.

Your line of thoughts is as follows:

The bend of the flight tube and the magnetic filed make so that only ions with appropriate speed and mass can reach the detector. However, the speed to which the ions are accelerated by the voltage on the accelerator plate also depends on the particles' mass. Thus the value of the accelerating voltage determines particles of what mass will be counted by the detector.

You are able to examine the system and take some measurements. You find that the coil that generates the magnetic field is 1-m long, has 2000 turns and the current in the coil is 80A; the radius of the 90-degree bend of the flight tube is 1.25 m; the acceleration voltage is being set by the operator.

Now, all you need is to plot the dependence of the acceleration voltage on the isotope atomic mass.

With the calibration curve at hand, you measure several samples and determine the U-235 fraction. Now you can answer the main question: Is this facility being used for the purpose of making a nuclear weapon …

To complete the assignment:

1. Review the lecture notes on solenoid and moving charged particles in magnetic field.
2. Based on your general knowledge on the topic, answer the first 4 questions in the Task Report Questionnaire.
3. Write a MATLAB code that computes and plots the calibration curve, i.e., acceleration voltage as a function of the ion's **atomic mass**. The ion's atomic mass equals the total number of protons and neutrons in the particle's nucleus. The actual ion's mass is the product of its atomic mass and the unified atomic mass unit, that is 1.66×10^{-27} kg. **The calibration curve must display the voltage in kV for a range of atomic mass from 200 to 250.**
4. Print out your MATLAB code and calibration curve.
5. Examine the calibration curve and use the data cursor to obtain the voltage values needed to answer questions 5 to 7 in the Task Report Questionnaire.

6. Answer the last two questions in the Task Report Questionnaire.
7. Staple the printouts to this booklet and hand the paper in by the next class period.
8. **Show all your work, to receive full credit.**

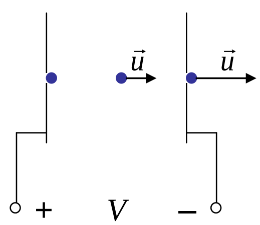
Task 5. The Nuclear Inspector

Questionnaire

Give brief but accurate and thorough explanation if necessary. Provide math expressions where needed to show how you've obtained the particular result.

1. What is the magnitude of the magnetic flux density produced by the solenoid? Provide both the general math expression and the particular value.

2. Assuming that all ions start at rest, provide the expression about their final speed when they are accelerated by the voltage on the accelerator plate. The following figure will help you understand the acceleration process. When the particle passes through the potential difference *V*, its potential energy is entirely converted into kinetic energy.



3. Provide the math expression for the Larmor radius and explain what actually the Larmor radius is.

4. Based on your answers to the above questions, write down an expression about the acceleration voltage in terms of particle's charge and mass, magnetic flux density, and Larmor radius.

5. From the calibration curve, what voltage is needed to measure the level of U-234?

6. From the calibration curve, what voltage is needed to measure the level of U-235?

7. From the calibration curve, what voltage is needed to measure the level of U-238?

8. Using your answer to question 6, compute the speed of the U-235 after acceleration.

9. What acceleration voltage is needed to measure the level of U-235 when the atoms are doubly ionized, i.e., two electrons have been knocked off the atom? You can answer this question by examining your answers to questions 4 and 6.

Task 6. A Friend in Need's a Friend Indeed

Your best friend is an ophthalmologist and has a small but lucrative practice. One day, you receive an urgent call from him. He sounds anxious and the reason is the following: An iron filing has gotten into the eye of one of his wealthiest patients. Sure enough, the gentleman has immediately arrived at the clinic. While your friend is getting ready for the routine operation of ferromagnetic foreign body extraction from the eye, he suddenly discovers that the hand-held electromagnet doesn't work. Of course, his first thought is to ask you for help because you are a design engineer with a company producing capacitors, inductors and other electrical devices. You sense that the young ophthalmologist is about to push the panic button, therefore you take off on the spur of the moment.

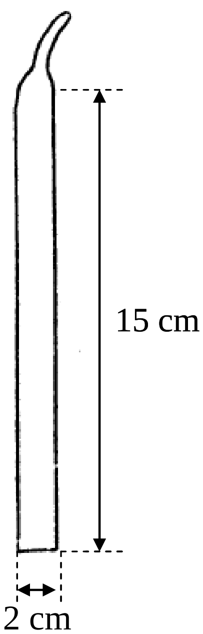


Fig. 1

The principle of operation of the hand-held electromagnet is the following: The ferromagnetic core has a long cylindrical part and a short thin tip, as shown in figure 1. Multiple turns of wire are wound on the cylindrical part of the core, thus forming a solenoid. The solenoid produces a strong magnetic field at the tip of the core. The field lines are shown in figure 2. In the region denoted by A in the figure, there is a strong gradient in the magnetic field intensity, which gives rise to the magnetic force of attraction experienced by a body with a non-zero magnetic moment.

At the clinic, you look up the properties of the magnet and find out that the core material has relative permeability of 1190, the length of the cylindrical part, where the windings are, is 15 cm, and the diameter is 2 cm. The magnet is fed by a 9-V battery and the magnetic flux density in the cylindrical part of the core is supposedly 9000 gauss.

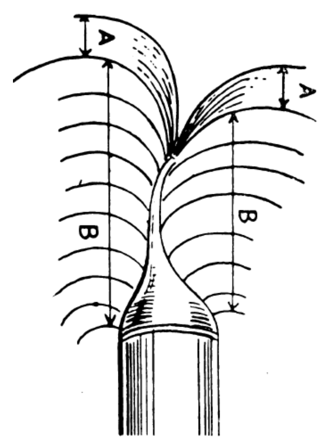


Fig. 2

You quickly discover that the electromagnet has burned out and you need to rewind the coil. Since you have expected something of the kind, you've brought a pocketful resistors and a spool of enamel copper wire ($\sigma = 5.7\times10^{7}$ S/m). The wire has a diameter of 0.2 mm. This diameter accounts for both the copper and the enamel layer. The diameter of the copper only is 0.18 mm. The maximum permissible current density in the wire is 2.55 A/mm^2.

The maximum permissible current through the copper wire is

$$I_{\max} = \int_S J_{\max}\, ds = J_{\max}\pi\left(\frac{d_{copper}}{2}\right)^2 = 2.55\times10^{6}\times\left(9\times10^{-5}\right)^2 = 0.0649 \approx 65\,\text{mA}$$

You keep in mind that the actual current through the coil must never exceed 65 mA!

You remove the old burned out winding. Then you quickly make some calculations and figure out that one layer of winding using the available wire won't work. Therefore, you wind two layers of winding.

In less than 30 minutes the hand-held electromagnet is ready for use. Your friend, the ophthalmologist, has a sigh of relief and the patient can stop taking pain-killers.

To complete the assignment:

1. Assume the solenoid is an ideal device and long enough so that the formula for the B-field inside an infinitely long solenoid is applicable.
2. Assume that the wire in the coil is being tightly wound, i.e., the turns of wire touch each other.
3. The B-field is given in gauss, which is not the SI unit for magnetic flux density. You must convert it to the SI unit Tesla.
4. Fill in the Questionnaire, print it out and hand the paper in by the next class period.
5. **Show all your work, to receive full credit.**

Task 6. A Friend in Need's a Friend Indeed

Questionnaire

Give brief but accurate and thorough explanation if necessary. Provide math expressions where needed.

1. What is the number of turns that can be tightly wound in **<u>one</u>** layer of winding? Give the math expression and the value.

2. What current through the coil is **needed** in order to get the right magnetic flux density when **<u>one</u>** layer of winding is used? Give the math expression and the value.

3. Why **<u>one</u>** layer of winding wouldn't work? Prove the correctness of your answer by providing values of the quantities involved.

4. What is the number of turns that can be tightly wound in **<u>two</u>** layers of winding?

5. What current through the coil is **needed** in order to get the right magnetic flux density when **two** layers of winding are used? Give the math expression and the value.

6. What is the length of the wire that has been wound in **two** layers of winding? Give the math expression and the value. ***Note:*** Neglect the thickness of the wire.

7. What is the total resistance of the wire in **two** layers of winding? Give the math expression and the value.

8. What current would be flowing through the coil (two layers of winding) when it is powered by the 9-V battery? Give the math expression and the value.

9. Does your answer to question 8 present a problem? If yes, how are you going to fix it? *Note:* Your friend has stocked up with 9-V batteries only.